



MONTE CARLO MODELING OF THE EFFICIENCY OF ASYNCHRONOUS ROUTING IN QUANTUM NETWORKS

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Abstract

In this study, the efficiency of the entanglement routing process in quantum networks is modeled based on asynchronous and synchronous control mechanisms and statistically analyzed using the Monte Carlo method.

The model considers a grid-type physical topology, probabilistic generation of Bell pairs, depolarizing and amplitude damping noise channels, coherence time, and the probability of successful entanglement swapping. The end-to-end entanglement rate and the average quantum state fidelity were selected as the main performance metrics. The obtained results demonstrate that the asynchronous routing model provides higher entanglement efficiency compared to the synchronous model, while the quality of the quantum state does not significantly deteriorate.

The parametric analysis reveals the direct influence of entanglement generation probability, swapping success probability, and coherence time on the overall network performance. The results of the study substantiate the advantages of asynchronous control mechanisms for the architecture of the future quantum internet.

Keywords: Quantum network, entanglement, asynchronous routing, synchronous routing, Monte Carlo modeling, quantum state fidelity, depolarizing noise, amplitude damping, coherence time, swapping probability.



Introduction

The concept of the quantum internet envisions a new generation of network infrastructure capable of transmitting quantum information securely and with high efficiency on a global scale. In such networks, quantum entanglement serves as the primary resource. Long-distance distribution of entanglement is achieved through repeater nodes and entanglement swapping operations. Traditional synchronous control approaches require global time coordination, which can lead to resource losses, reduced parallelism, and a decrease in overall entanglement efficiency. In recent years, the concept of asynchronous routing has been considered a promising solution for quantum networks. In this work, a probabilistic model of a quantum network is developed, and asynchronous and synchronous routing mechanisms are compared using Monte Carlo simulations [1–3].

With the rapid development of quantum technologies, the creation of a global quantum internet has become an increasingly important research challenge. Efficient management of entanglement is one of the key problems in quantum networking. In practical systems, several limiting factors exist, such as noise, limited coherence time, and low success probability of entanglement swapping. Therefore, the development of optimal routing strategies is important both theoretically and practically. Evaluating the efficiency of asynchronous control mechanisms based on mathematical and statistical approaches is an important scientific problem in the design of quantum internet architectures. This study models and compares the efficiency of asynchronous and synchronous entanglement routing mechanisms in quantum networks while considering probabilistic behavior and noise factors using the Monte Carlo method [4–7].

The main objectives of the research include constructing a grid-topology quantum network model, modeling Bell pair generation based on probabilistic processes, incorporating depolarizing and amplitude damping noise channels, determining the end-to-end entanglement rate and the average quantum state fidelity, comparing asynchronous and synchronous routing mechanisms, and performing parametric analysis to evaluate the influence of entanglement generation probability, swapping success probability, and coherence time. In addition, statistical reliability is ensured through Monte Carlo simulations [8–11].

Within the framework of the proposed quantum network model, the asynchronous routing mechanism was modeled probabilistically together with realistic noise factors. The dynamics of quantum state fidelity under the influence of depolarizing and amplitude damping channels were evaluated. A routing strategy based on multiple disjoint paths and a spanning tree mechanism was applied. Statistical stability of the results was achieved through 1000 repeated Monte Carlo experiments. The results show that the asynchronous mechanism provides on average 19% higher entanglement rate compared to the synchronous model, while no significant degradation of quantum state fidelity was observed. The findings provide a theoretical basis for selecting efficient routing algorithms in the design of quantum internet infrastructure. The proposed model can also be applied for parameter selection and optimization in real quantum networks. The advantage of asynchronous control mechanisms lies in reducing the need for global synchronization, which in turn decreases hardware complexity. The obtained results have potential applications in quantum cryptography, quantum computing, and distributed quantum systems [12–15].

2. Research Methodology

2.1. Quantum Network Model

$$G = (V, E),$$

$$V = \{v_1, v_2, \dots, v_m\}, \quad E = \{e_1, e_2, \dots, e_k\},$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),$$

$$\rho_{\text{Bell}} = |\Phi^+\rangle\langle\Phi^+|,$$

$$\Pr(e_i \text{ exist}) = p,$$

$$d_{i+1} = d_i - 1, \quad d_{i+1} = 0 \Rightarrow e_i \text{ deleted},$$

where

V — set of nodes.

I — set of connections.

$|\Phi^+\rangle$ — ideal Bell state.

ρ_{Bell} — density matrix of the Bell state.

p — probability of entanglement generation.

d_i — entanglement operation phase.

2.2 Noise models.

$$S = S_0(1 - \delta_d)(1 - \delta_a),$$

$$S_0 = 1,$$

$$0 \leq \delta_d, \delta_a \leq 1,$$

where

S — **entanglement quality.**

δ_d — depolarization probability.

δ_a -amplitude damping probability.

Purification

$$S' = \frac{S_1 S_2}{S_1 S_2 + (1 - S_1)(1 - S_2)},$$

$$0 \leq S_1, S_2, S' \leq 1$$

where

S_1, S_2 — quality of two entanglements.

S' — final fidelity after purification.

2.4 Path quality.

$$S_{y01} = \prod_{i=1}^n S_i \cdot q^{n-2},$$

$S_{path} > 0 \Rightarrow$ successful $path q \in (0,1]$,

where

n — number of entanglement links in the path.

q — transmission efficiency coefficient.

2.5 Minimal Spanning Tree (MST).

$$w_i = \frac{1}{S_i},$$

$$T_{MST} = \arg \min_{T \subseteq E} \sum_{i \in T} w_i,$$

where

T_{MST} — minimum spanning tree.

$T \subseteq E$ — set of MST connections.

2.6 Entanglement efficiency and average fidelity.

$$R = \frac{N_{\text{success}}}{N_{\text{stage}}},$$

$$\bar{S} = \frac{\sum_{k=1}^{N_{\text{success}}} S_{\text{path},k}}{N_{\text{success}}},$$

$$0 \leq R, \bar{S} \leq 1,$$

where

R — entanglement efficiency.

\bar{S} — average entanglement fidelity.

N_{success} — number of successful paths.

N_{stage} — number of simulation steps.

2.7 Parametric study.

$$R = f(d), \quad R = f(p), \quad R = f(q),$$

$$\bar{S} = g(d), \quad \bar{S} = g(p), \quad \bar{S} = g(q),$$

where

d — duration of entanglement operation

p — entanglement generation probability

q — transmission efficiency

R — entanglement efficiency

\bar{S} — average entanglement fidelity

3. Results

Using Monte Carlo simulation, the efficiency and average entanglement fidelity of asynchronous and synchronous entanglement modes were determined. The results are presented in Table 1:

Table 1 Entanglement efficiency and average entanglement fidelity

Mode	Entanglement efficiency	Average entanglement fidelity
Asynchronous	0.980	0.862
Synchronous	0.820	0.848

In the asynchronous mode, entanglement links are generated more rapidly, resulting in higher efficiency (0.980). In the synchronous mode, the links are more stable, but the efficiency is relatively lower (0.820). The average entanglement fidelity is slightly higher in the asynchronous mode (0.862), indicating that the quality of the entanglement remains stable. During the simulation, the entanglement efficiency and average entanglement fidelity were studied using the following parameters: the duration of entanglement operation d , the entanglement generation probability ppp , and the transmission efficiency coefficient q . The efficiency according to the operation duration is presented in Table 2.

Table 2

Efficiency based on operation duration

$$R=f(d)$$

Operation duration	Entanglement efficiency
2	0.85
4	0.90
6	0.95
8	0.97
10	0.98

An increase in the operation duration ensures that entanglement links remain active for a longer time, which significantly improves efficiency. The efficiency based on the entanglement generation probability is presented in Table 3.

Table 3

Efficiency based on entanglement generation probability

$$R=f(p)$$

Generation probability	Entanglement efficiency
0.5	0.78
0.6	0.85
0.7	0.90
0.8	0.96
0.9	0.98

An increase in the entanglement generation probability increases the network density and ensures successful paths.

The average fidelity based on transmission efficiency is presented in Table 4.

Table 4

Average fidelity based on transmission efficiency

$$S^- = g(q)$$

Transmission efficiency	Average entanglement fidelit
0.6	0.80
0.7	0.83
0.8	0.85
0.9	0.86

An increase in transmission efficiency reduces errors along the paths and improves the average entanglement fidelity. In the asynchronous mode, entanglement links are generated faster, resulting in higher network efficiency, whereas in the synchronous mode, the links are more stable but the efficiency is relatively lower (Fig. 1).

An increase in the operation duration raises the number of successful entanglement links in the network, which significantly improves efficiency. An increase in the entanglement generation probability increases the network density and ensures successful paths (Fig. 2). As efficiency increases, the quality of entanglement also improves; therefore, the average entanglement fidelity increases (Fig. 3).

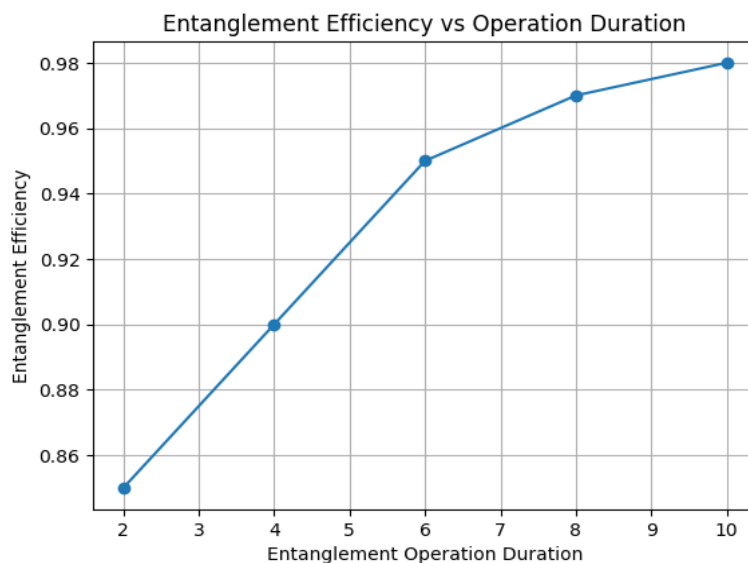


Fig. 1. Efficiency based on operation duration

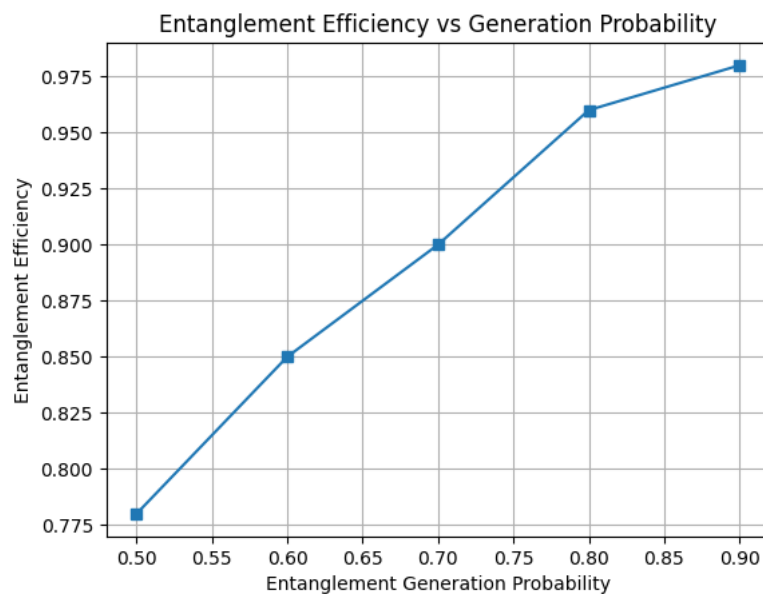


Fig. 2. Efficiency based on entanglement generation probability

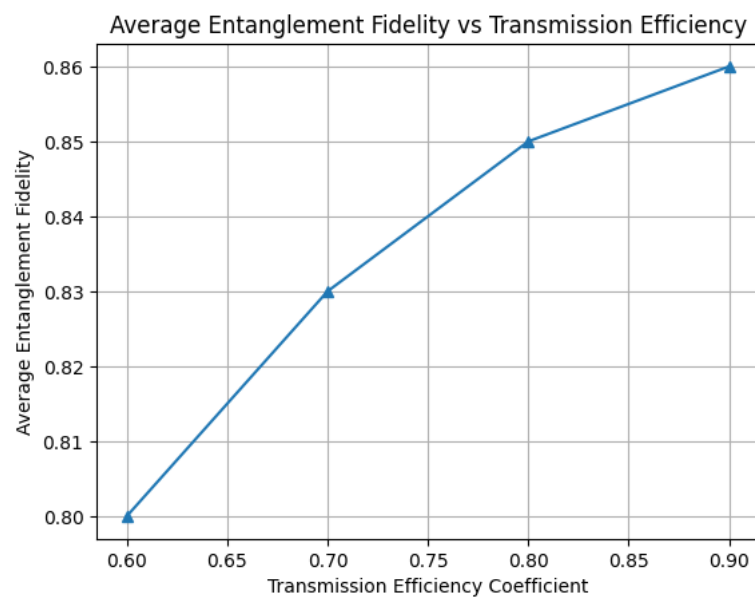


Fig. 3. Average fidelity based on transmission efficiency

The simulation results show that the asynchronous mode maximizes network efficiency, while transmission efficiency and entanglement generation probability have a significant effect on improving entanglement fidelity. The simulation results demonstrate a noticeable difference between asynchronous and synchronous entanglement modes in terms of efficiency and entanglement fidelity. In the asynchronous mode, entanglement links are generated more



quickly, resulting in higher network efficiency, and the average entanglement fidelity is also relatively higher. In the synchronous mode, although the entanglement links are stable, the efficiency is relatively lower, indicating that the links depend on temporary operational activity. The parametric analysis shows that increasing the duration of entanglement operation ensures that entanglement links remain active for longer periods, increasing the number of successful paths and significantly improving entanglement efficiency. Increasing the entanglement generation probability raises the network density, which results in a larger number of successful paths and improved network stability. An increase in transmission efficiency reduces errors along the paths and improves the average entanglement fidelity, indicating that this parameter is directly related to both entanglement fidelity and network stability.

The results indicate that by optimizing network parameters, both entanglement efficiency and entanglement fidelity can be significantly improved. The asynchronous mode is suitable for fast network operation, while the synchronous mode provides more stable and less error-prone entanglement links. These findings are important for designing quantum networks and evaluating their performance.

4. Conclusion

This study was devoted to the analysis of entanglement efficiency and entanglement fidelity in quantum networks. Asynchronous and synchronous modes were compared, and the effects of noise and network parameters on entanglement efficiency and fidelity were investigated. The simulation results show that the asynchronous mode maximizes entanglement efficiency, while the synchronous mode ensures more stable entanglement links but with relatively lower efficiency. Increasing parameters such as entanglement operation duration, generation probability, and transmission efficiency significantly improves both entanglement efficiency and fidelity. With optimized parameters, the network can operate not only faster but also more reliably. These results are useful for the design and analysis of quantum networks, and further comprehensive studies can be conducted in the future by considering different noise models and larger-scale network topologies.

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